Let $I = \int_{-\pi}^{\pi} e^{\sin(x)}\,dx$

1. Approximate I accurate within 0.01 using a right sum

   The question here is: how many subintervals n should I use in my right sum to *guarantee* that R_n is within .01 of I.

   That is,
   Question: For what n are we guaranteed that $|I - R_n| \leq .01$.?

   We *want* $|I - R_n| \leq .01$.

   We *know* from Theorem 3 that $|I - R_n| \leq \dfrac{K_1(b-a)^2}{2n}$.

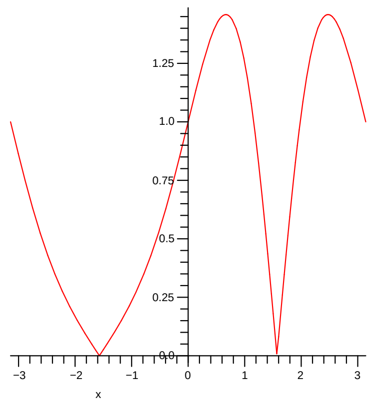
   Need to find: n so that $\dfrac{K_1(b-a)^2}{2n} \leq .01$. Then we'll have that

   $$|I - R_n| \leq \frac{K_1(b-a)^2}{2n} \leq .01,$$

   or in other words, we'll have that, as desired, $|I - R_n| \leq .01$.

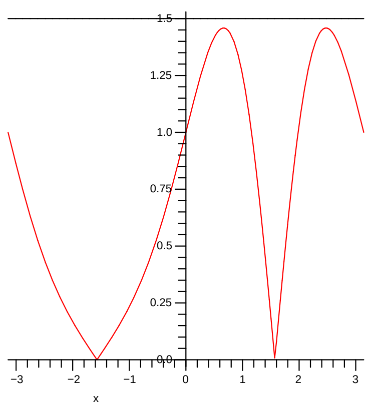
   Our first step must be to find K_1. In order to do that, we graph $|f'(x)|$ over $[-\pi, \pi]$ on Maple.

```
f:= x -> exp(sin(x));
plot(abs(diff(f(x), x)), x=-Pi..Pi);
```



K_1 needs to be greater than $|f'(x)|$ on the interval $[-\pi, \pi]$, so I look at the graph of $|f'(x)|$ above and choose a number I think looks higher than the graph ever gets. It looks to me as if $|f'(x)| \leq 1.5$. I don't just assume I'm right, of course – I plot the horizontal line $y = 1.5$ together with $|f'(x)|$ to make sure that the horizontal line is always higher than the absolute value of the derivative:

```
plot([1.5,abs(diff(f(x),x))],x=-Pi..Pi, color=[black,red]);
```



Now that I know that for all x in $[-\pi, \pi]$, $|f'(x)| \leq 1.5$, I know that I can choose to use

$$K_1 = 1.5.$$

Notice that I could also choose to use any number larger than 1.5 as K_1. Doing so would produce more subintervals in the end. I could also choose to try to hone in a little more closely on a smaller K_1 by changing the constant in the above graph, and changing the interval you're graphing over to zoom in on the maxima. It turns out that about the smallest value of K_1 you can reasonably use is $K_1 = 1.46$.

It's time to find the number of subintervals n so that $\dfrac{K_1(b-a)^2}{2n} \leq .01$:

$$\begin{aligned}
\frac{K_1(b-a)^2}{2n} &\leq .01 \\
\frac{1.5[\pi-(-\pi)]^2}{2n} &\leq .01 \\
\frac{1.5(2\pi)^2}{2n} &\leq .01 \\
\frac{6\pi^2}{2n} &\leq \frac{1}{100} \\
300\pi^2 &\leq n \\
2960.88 &\leq n \\
2961 &= n
\end{aligned}$$

Therefore I is within .01 of R_{2961}.

I use Maple to find R_{2961}:

```
with(student):
rightsum(f(x), x=-Pi..Pi, 2961);
evalf(%);
```

And I end up with $R_{2961} = 7.954926525$, so I is within .01 of 7.955.

2. Approximate $I = \int_{-\pi}^{\pi} e^{\sin(x)}\,dx$ within 0.001 using a midpoint sum

   Once again, the question is how many subintervals do I need, this time in order to guarantee that M_n is within .001 of I?

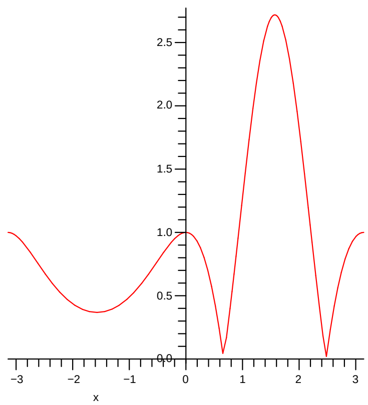
   We *want* $|I - M_n| \leq .001$.

   We *know* $|I - M_n| \leq \dfrac{K_2(b-a)^3}{24n^2}$.

   Need to find: n so that $\dfrac{K_2(b-a)^3}{24n^2} \leq .001$.

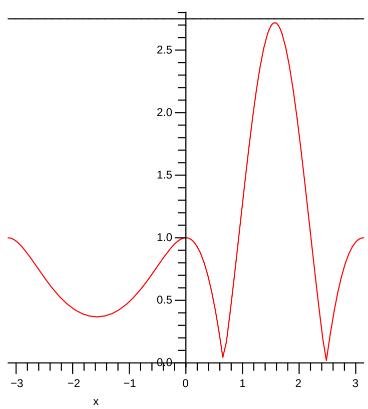
   In order to find a good choice for K_2, we graph $|f''(x)|$ over $[-\pi, \pi]$ on Maple:

```
plot(abs(diff(f(x),x,x)), x=-Pi..Pi);
```



It looks like $|f''(x)|$ is always less than 2.75 or so. I graph both $y = 2.75$ and the absolute value of the second derivative to see if I'm right:

```
plot([2.75,abs(diff(f(x),x,x))], x=-Pi..Pi, color=[black,red]);
```



I don't know if you can see it at this scale, but indeed the horizontal line at $y = 2.75$ is always above the absolute value of the second derivative on the interval $[-\pi, \pi]$, and so I can use

$$K_2 = 2.75.$$

And now I'm ready to find the number of subintervals I need:

$$
\begin{aligned}
\frac{K_2(b-a)^3}{24n^2} &\leq .001 \\
\frac{2.75(2\pi)^3}{24n^2} &\leq .001 \\
\frac{22\pi^3}{24n^2} &\leq \frac{1}{1000} \\
\frac{11000\pi^3}{12} &\leq n^2 \\
28422.42 &\leq n^2 \\
168.5895023 &\leq n \\
n &= 169
\end{aligned}
$$

Therefore I is within .001 of M_{169}.

I use Maple to find M_{169}:

```
middlesum(f(x),x=-Pi..Pi, 169);
evalf(%);
```

And I end up with $M_{169} = 7.954926523$, so I is within .001 of 7.9549.