Taylor's Theorem:

Let $f(x)$ be a function which is repeatedly differentiable on an interval I containing x_0. Suppose $P_n(x)$ is the n-th order Taylor polynomial based at x_0. Further suppose K_{n+1} is a bound for $|f^{(n+1)}(x)|$ on I. That is,

$$|f^{(n+1)}(x)| \leq K_{n+1} \text{ for all } x \in I$$

Then for all $x \in I$,

$$|f(x) - P_n(x)| \leq \frac{K_{n+1}}{(n+1)!}|x - x_0|^{n+1}$$

Let $f(x) = \cos(x)$ and let $x_0 = \frac{\pi}{2}$.

1. Find $P_3(x)$
2. Verify your answer by graphing $P_3(x)$ and $f(x)$ on the same set of axes with $-\pi/2 \leq x \leq 3\pi/2$
3. Use $P_3(2)$ to approximate $\cos(2)$
4. How accurate is your answer?
5. Find a value of n so that $P_n(2)$ approximates $\cos(2)$ accurate within 10^{-5}.