Determining Convergence - Important Reminders

Consider $I = \int_a^\infty f(x)\,dx$.

1. There is a huge distinction between $f(x)$ converging – that is, $\lim_{x\to\infty} f(x)$ being finite – and $I = \int_a^\infty f(x)\,dx$ converging. Just because you can find $\lim_{x\to\infty} f(x)$, and it's a finite number, does **not** mean that $\int_a^\infty f(x)\,dx$ will be finite.
2. In fact, if $\lim_{x\to\infty} f(x)$ exists but is not 0, I diverges! No need to investigate any further.
3. If $\lim_{x\to\infty} f(x) = 0$, I may converge or it may diverge – you must investigate further.

Determine whether each of the following improper integrals converges or diverges.

1. $\displaystyle\int_2^\infty \frac{1}{\sqrt{x}-2}\,dx$

2. $\displaystyle\int_2^\infty \frac{4}{x^3+2}\,dx$

3. $\displaystyle\int_0^1 \frac{2}{\sqrt{x}+x^2}\,dx$

4. $\displaystyle\int_0^\infty \frac{2}{\sqrt{x}+x^2}\,dx$