Goal: When faced with an improper integral that we can't evaluate simply by taking limits and antidifferentiating, how do we determine its convergence and approximate it if possible?

1. **Determine whether or not the improper integral converges when we cannot find an antiderivative.**

   Try to find a useful comparison-integral whose convergence or lack thereof we already know or can easily figure out.

2. **If our improper integral *does* converge, is there a way to approximate the value of it?**

   (a) Replace the improper integral with a proper one. The error from dropping the *tail* will equal the *tail* . Bound that by using a larger comparison-integral.

   (b) Use one of our approximation techniques (left sum, midpoint sum, etc) to estimate the new proper integral. This will also create an error; bound it using the formulas from Section 6.2.

   (c) Bound the error by adding the two error bounds.

Consider the improper integral $I = \int_1^\infty \frac{5}{x^3 + e^x}\,dx$

1. Determine whether I converges or diverges.

2. If I converges, find a definite integral which is within .005 of the improper integral. If I diverges, find a t so that $\int_a^t f(x)\,dx > 500$.

Show that $\int_1^\infty \frac{x}{x^5 + x^2 + 2}\, dx$ converges, and approximate its value accurate within 0.0001.