1. The nth Taylor polynomial for $f(x)$ based at $x = x_0$ is

$$\begin{aligned} P_n(x) &= f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 \\ &\quad + \frac{f'''(x_0)}{3!}(x - x_0)^3 \cdots + \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n \end{aligned}$$

2. The idea behind Taylor polynomials approximating a function $f(x)$ is to focus on how f behaves at *one point* x_0. We match not only the y-values at x_0, but also the slopes (the first derivative), the concavity (the second derivative), and however many more derivatives we choose – n is the number of derivatives we're choosing to match.

3. It seems that the higher n is, the better an approximation $P_n(x)$ gives.

4. We can approximate $\cos(x)$ near $x = 0$ by

$$\cos(x) \approx 1 - x^2/2! + x^4/4! - x^6/6!$$

and near $x = 2\pi$ by

$$\cos(x) \approx 1 - (x - 2\pi)^2/2! + (x - 2\pi)^4/4! - (x - 2\pi)^6/6!$$

The first 6 derivatives of $\cos(x)$, evaluated at $x = 0$:

$f(x) = \cos(x)$	$f(0) = 1$
$f'(x) = -\sin(x)$	$f'(0) = 0$
$f''(x) = -\cos(x)$	$f''(0) = -1$
$f'''(x) = \sin(x)$	$f'''(0) = 0$
$f^4(x) = \cos(x)$	$f^4(0) = 1$
$f^5(x) = -\sin(x)$	$f^5(0) = 0$
$f^6(x) = -\cos(x)$	$f^6(0) = -1$