

Let $f(x) = e^x$.

Let $P_n(x)$ be the n th order Taylor polynomial for $f(x)$ at $x_0 = 0$.

1. (a) Find $P_n(x)$ for $n = 0, 1, 2, 3, 4$. (Note that you can find all of these in one fell swoop if you start with the work for $P_4(x)$.)
(b) For $n = 0, 1, 2, 3, 4$, check how well $P_n(x)$ approximates $f(x)$ by graphing $P_n(x)$ and $f(x)$ on the same set of axes. You can experiment with what domain you use, but I've found $x = -3$ to $x = 3$ to be a good choice.
Remember: In Maple, we type in `exp(x)` rather than `e^x`.
2. Use $P_3(x)$ to find an approximation for $e^{1/2}$ and for e^2 . Will these be larger or smaller than the actual value of $e^{1/2}$ and e^2 ? From the graphs, do they look like good or bad approximations?

Let $f(x) = \sin(x)$ and let $P_5(x)$ be the 5th order Taylor polynomial for $f(x)$ at $x_0 = \pi$.

1. Find $P_5(x)$
2. Verify your answer by graphing $P_5(x)$ and $f(x)$ on the same set of axes.
3. Use $P_5(x)$ to find an approximation for $\sin(4)$ and for $\sin(6)$. Will these be larger or smaller than the actual value of $\sin(6)$? How good approximations are they?