Time: 5:30-8:30pm, Thursday 2/28
Location: SC 1313, SC 1314
What it Covers: All material through Friday 2/22
That is, Section 9.1-9.5 (excluding surface area), 10.1-10.5, and 11.1

- I will aim for the exam to be a 1 1/2 - 2 hour exam, but plan on it taking up to 3 hours.
- You may have one sheet of notes, consisting of **handwritten** (by you) notes on **one** side of a standard 8 1/2 x 11 sheet of paper. You may put any definitions, theorems, or examples you like on it.
- **During the exam, all phones, tablet computers, laptops must be off and away.**
- You will not have access to phone or computer, so bring a separate calculator.
- You may **not** use your calculator to solve equations, do any Calculus, or to graph.
- Anything covered in your reading, homework, or this study guide is fair game.
- The exam can not cover everything, but the main point is that you learn it well, not that I test it thoroughly.
- Through studying, you should be attaining a deeper and more connected understanding of the material, combining ideas we may not have combined in class, so don't expect exam questions to be like homework questions.
- Below, I have a list of some topics you should know. It is not comprehensive, but I tried to design it in a way that points out some connections between things we've covered.
- As you know doubt have learned by now, the more problems you work through the better prepared you will be.
- Following that list, I have included some sample problems to give you a sense of the style of question that may be on the exam. These do not cover every topic that maybe covered on the exam, so doing these should not constitute the entirety of your studying.
- There are additional problems on WeBWorK, under the name Study Guide 1.

Topics you should know:

1. What polar coordinates are, how to plot polar points, and how you convert from polar to rectangular, or from rectangular to polar.
2. What parametric equations (in two and three dimensions) **are**
3. How to sketch a fairly simple parametrically defined curve, polar curve, or vector-valued function by plotting points.
4. Clues that help match graphs of parametrically defined curves, vector-valued functions, and polar curves to their equations.
5. What a vector-valued function is; the relationship between parametric equations and vector-valued functions.

6. How to find parametric equations, or vector-valued functions, for a line in the plane or in space; for portions of a circle of any radius, or of an ellipse, in the plane if you know the long and short dimensions (the major and minor axes), whether centered at the origin or not, whether traversed clock- or counter-clockwise, and whether starting at the top, bottom, leftmost point or rightmost point.
7. Slope of line tangent to a plane curve given parametrically
8. How to find where a plane curve has horizontal tangent lines; where it has vertical tangent lines
9. How to find velocity and speed if an object's position is given parametrically. *Through Friday, we will have only covered how to do this in the plane, but in addition to our covering it a it on Monday, extending to three dimensions is the sort of extension I would expect you to be able to do.*
10. How to convert a polar equation to a rectangular (xy) equation.
11. How to find where two polar curves intersect, or when one polar equation has completed one loop, or one leaf.
12. Area enclosed by a curve given parametrically
13. Area enclosed by a curve, or a portion of a curve (like a leaf or inner loop), of a polar region.
14. Arclength of a curve, whether it is given parametrically (in $\mathbb{R}^2$ or $\mathbb{R}^3$), in polar form, or as a vector-valued function.
15. All the basics of vectors: what they are, how to find the displacement vector from a point A to a point B, magnitude, the dot/scalar product (especially that it's a scalar) and the cross/vector-product (especially that it's a vector). This includes the various properties (additive, distributive, etc) associated with the dot product, and the more common ones associated with the cross-product.
16. $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = \|\vec{\mathbf{a}}\|\|\vec{\mathbf{b}}\|\cos(\theta)$ and $\|\vec{\mathbf{a}} \times \vec{\mathbf{b}}\| = \|\vec{\mathbf{a}}\|\|\vec{\mathbf{b}}\|\sin(\theta)$.
17. The triangle inequality
18. How to find $\text{comp}_{\vec{\mathbf{b}}}\vec{\mathbf{a}}$ and $\overrightarrow{\mathbf{proj}}_{\vec{\mathbf{b}}}\vec{\mathbf{a}}$ (and, of course, what these *are*, including which is a vector and which is a scalar). Also, how to find the *orthogonal component* and the *orthogonal projection.*
19. How to find a vector that's orthogonal to two given vectors
20. How to immediately know a direction vector for a line defined parametrically (or with a vector-valued function), and the normal vector for a plane.
21. How to find the distance between a point and a line (there are at least two ways, one involving cross-product, the other dot-product; be sure to know at least one.)
22. How to find the distance from a point to a plane, or between two parallel planes.
23. How to find the equation of a plane, given a point and a normal vector (or three points, or two lines on the plane, etc) and conversely, how to find a point on a plane or its normal vector, given its equation.

24. How to decide whether two lines intersect, are parallel, or are skew
25. How to find the intersection of a line and a plane
26. How to find the intersection of two planes

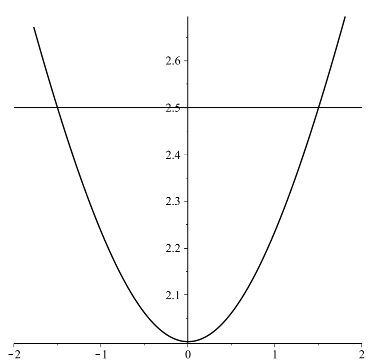
Sample problems:

1. Suppose that the position of one particle at time t is given by
$$x_1(t) = \sqrt{3}\cos(t) \qquad y_1(t) = 2\sin(t), 0 \le t \le 2\pi$$
and the position of a second particle at time t is given by
$$x_2(t) = 2\sin(t) \qquad y_2(t) = \sqrt{3}\sin(t), 0 \le t \le 2\pi.$$
Do the two particle's paths cross? If so, where? Do the two particles collide?

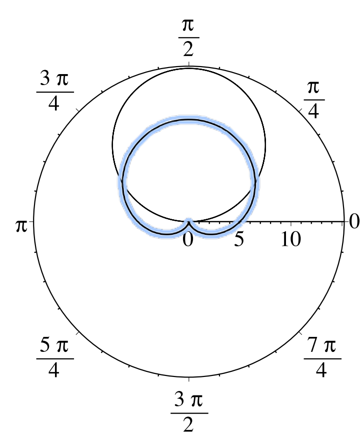
2. Assuming that the units are the same for both cars' velocity vectors, which is traveling faster: a car whose velocity vector is $25\,\overrightarrow{\mathbf{i}} + 30\,\overrightarrow{\mathbf{j}}$, or a car whose velocity vector is $40\,\overrightarrow{\mathbf{i}}$? At what speed is the faster car traveling?

3. Find an equation of the largest sphere with center $(8, 10, 5)$ that is completely contained in the first octant. (It's okay if it touches one or more of the coordinate planes.)

4. Find the area of the region shown below bounded by the parametric curve
$$x(t) = t - \frac{1}{t} \qquad y(t) = t + \frac{1}{t}$$
and the line $y = 2.5$.



Graph of $\big(x(t), y(t)\big)$



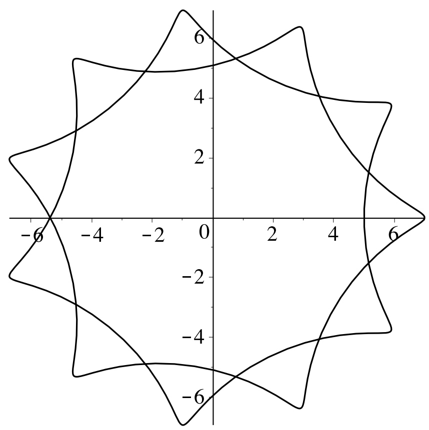
Graph of $r(\theta)$ and $R(\theta)$

5. Find the area of the region shown above that is outside $r(\theta) = 5 + 5\sin(\theta)$ but inside $R(\theta) = 15\sin(\theta)$.

6. Assume t is defined for all real numbers. Without using graphing technology, match each graph to one of the given pairs of parametric equations. Explain briefly how you reached each conclusion.

   Hint: As you look at the given parametric equations, think about the ranges of x and y, whether either or both variables will be symmetric, and whether either of both variables will be periodic.

A.



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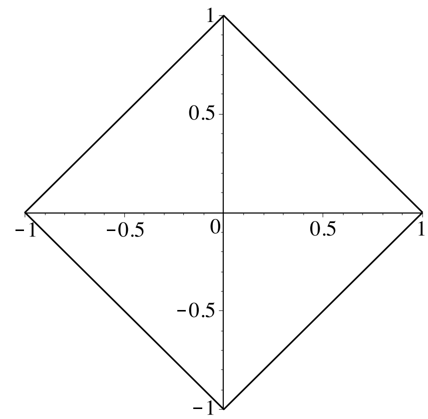

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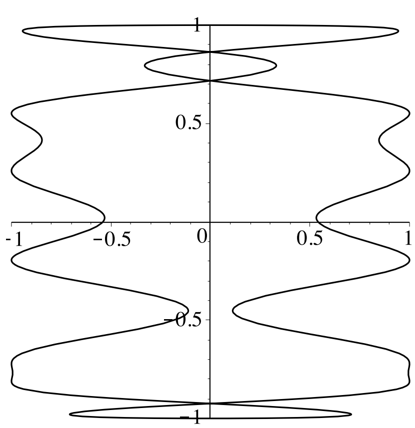

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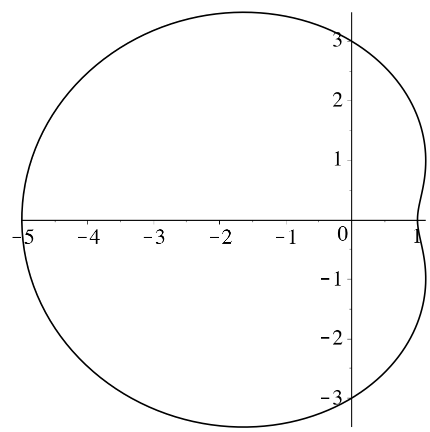

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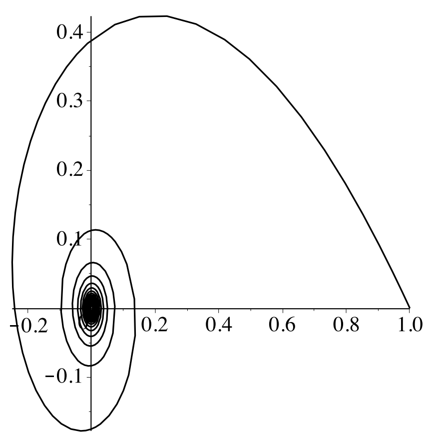


(a) $x(t) = \sin(t)\big(3 - 2\sin(t)\big)$
$y(t) = \cos(t)\big(3 - 2\sin(t)\big)$

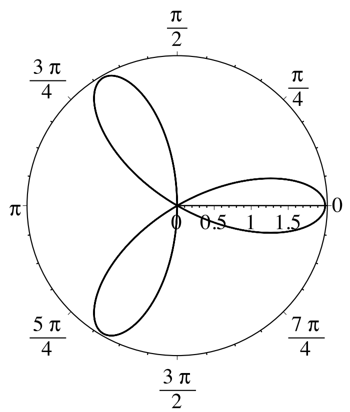
(b) $x(t) = \big|\cos(t)\big|\cos(t)$
$y(t) = \big|\sin(t)\big|\sin(t)$

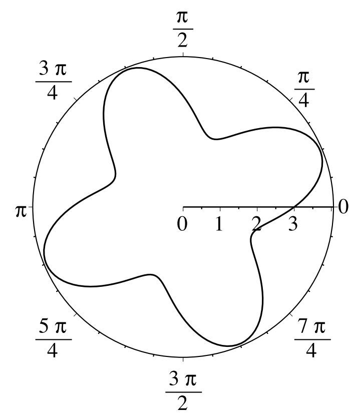
(c) $x(t) = 6\cos(t) + \cos(4.5t)$
$y(t) = 6\sin(t) - \sin(4.5t)$

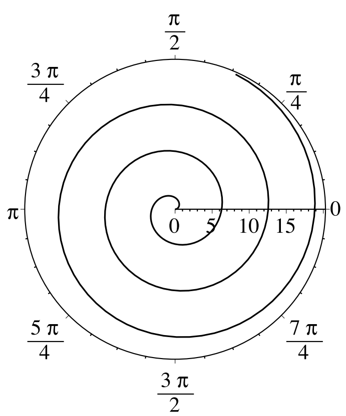
(d) $x(t) = \dfrac{1}{1+t^2}\cos(t^2)$
$y(t) = \dfrac{1}{1+t^2}\sin(t^2)$

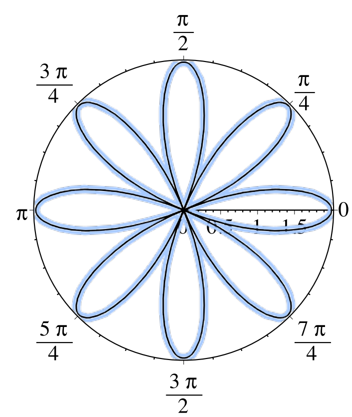
(e) $x(t) = \sin\big(t + \sin(7t)\big)$
$y(t) = \cos(t)$

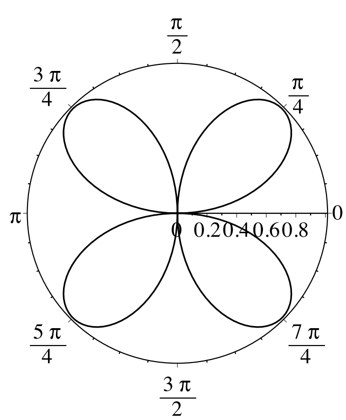
7. Without using graphing technology, match each graph to one of the given polar equations. Explain briefly how you reached each conclusion.

A. 

B. 

C. 

D. 

E. 

(a) $r(\theta) = \theta,\ \theta \geq 0$

(b) $r(\theta) = \sin(4\theta) + 3$

(c) $r(\theta) = \sin(2\theta)$

(d) $r(\theta) = 2\cos(3\theta)$

(e) $r(\theta) = 2\cos(4\theta)$

8. Consider the parametric equations

$$x(t) = 5\sin(t) \qquad y(t) = 4\cos^2(t).$$

(a) Working with the parametric equations, find all point(s) $P(x, y)$ (if any) on the curve where the tangent line is horizontal.

(b) Again working with the parametric equations, find all point(s) $Q(x, y)$ (if any) on the curve where the tangent line is vertical.

(c) Write the given pair of parametric equations in the form $y = f(x)$.
Write your result without any trig or inverse trig functions.

(d) What is the domain of the function found in Part (c)?

9. Decide whether the following statements are true or false. If a statement is true, briefly explain why; if it is false, give a counter-example.

   (a) For a given curve, there is exactly one set of parametric equations that describes it.
   (b) If f is periodic with fundamental period 2π, then exactly one copy of $r = f(\theta)$ is traced out with $0 \leq \theta \leq 2\pi$.
   (c) For any scalar k and any vector $\vec{\mathbf{v}}$, $\|k\vec{\mathbf{v}}\| = k\,\|\vec{\mathbf{v}}\|$
   (d) For all vectors $\vec{\mathbf{a}}$, $\vec{\mathbf{b}}$, $\left|\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}\right| < \|\vec{\mathbf{a}}\|\,\|\vec{\mathbf{b}}\|$
   (e) If $\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \vec{\mathbf{a}} \times \vec{\mathbf{c}}$, then $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ are parallel
   (f) $(\vec{\mathbf{i}} \times \vec{\mathbf{k}}) \cdot \vec{\mathbf{k}} = \vec{\mathbf{i}} \cdot (\vec{\mathbf{j}} \times \vec{\mathbf{k}})$
   (g) The value of $\vec{\mathbf{v}} \cdot (\vec{\mathbf{v}} \times \vec{\mathbf{w}})$ is always zero
   (h) If $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$ are any two vectors, then $\|\vec{\mathbf{v}} + \vec{\mathbf{w}}\| = \|v\| + \|w\|$
   (i) Two planes are parallel if and only if their normal vectors are parallel

10. A high school basketball gymnasium is 15 m high, 30 m wide, and 45m long. For a half-time stunt, the cheerleaders want to attach two pieces of rope: one from each of the two ceiling corners behind and above one basket to the diagonally opposite corners of the gym floor. Choose to place the origin in the center of the gym's floor (and assume that this is also the center of the basketball court itself). Further assume that the ropes are attached tightly enough to form straight lines.

    (a) Find parametric equations for each of the line segments formed by the ropes. Include what interval t should be in to give **only** the ropes.

    (b) What is the angle (in radians) made by the ropes as they cross?

    (c) The two free throw lines are each roughly 7 meters from the center of the court (in the long direction), and are aligned with the center (that is, the midpoints of the two free throw lines and the center of the court form a line which is parallel to the long side of the court). How far is the midpoint of one of the free throw lines from either of the ropes?

    (d) Find an equation for the plane formed by the ropes.

    (e) How far is the center of the gym's floor from this plane?